

Practise

For help with questions 1 to 4, see Example 1.

- Sketch graphs of all three relations on the same set of axes. Label the x-intercepts, vertex, and axis of symmetry for each parabola. Then, describe the similarities and differences between the graphs.

- $y = (x + 3)(x - 1)$.
- $y = 2(x + 3)(x - 1)$
- $y = -2(x + 3)(x - 1)$

- Sketch graphs of all three relations on the same set of axes. Label the x-intercepts, vertex, and axis of symmetry for each parabola. Then, describe the similarities and differences between the graphs.

- $y = (x - 4)(x - 8)$
- $y = \frac{1}{2}(x - 4)(x - 8)$
- $y = \frac{1}{4}(x - 4)(x - 8)$

- Sketch each parabola. Label the x-intercepts and vertex.

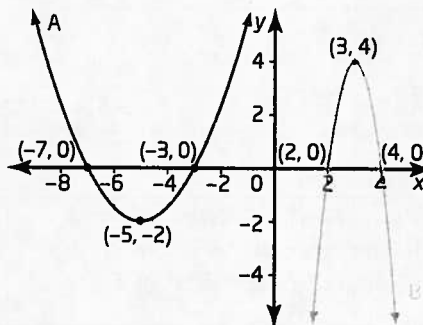
- $y = (x - 6)(x - 2)$
- $y = -(x + 3)(x + 7)$
- $y = 2(x - 3)(x + 2)$
- $y = -2(x - 4)(x + 2)$

- Sketch each parabola. Label the x-intercepts and vertex.

- $y = 3x(x + 2)$
- $y = \left(x + \frac{1}{2}\right)\left(x - \frac{7}{4}\right)$
- $y = -0.2(x - 4)(x + 10)$
- $y = \frac{2}{3}(x - 6)(x + 9)$
- $y = (x + 3.5)(x - 3.5)$
- $y = -\frac{1}{3}(x - 0.5)(x - 0.1)$

For help with question 5, see Example 2.

- Determine an equation in the form $y = a(x - r)(x - s)$ to represent each parabola. Consider the vertex and x-intercepts.



Connect and Apply

- You investigated the graphs of $y = (x - h)^2$ in Section 4.3. Consider the quadratic relation $y = (x - 5)^2$.
 - Write the coordinates of the vertex of the parabola.
 - How many x-intercepts does the parabola have?
 - Rewrite the equation in the form $y = a(x - r)(x - s)$.
- A parabola has equation $y = (x + 2)^2$.
 - Write its x-intercepts.
 - Determine the coordinates of its vertex.
- The predicted flight path of a toy rocket is defined by the relation $h = -2(d - 3)(d - 15)$, where d is the horizontal distance, in metres, from a safety wall, and h is the height, in metres, above the ground.
 - Sketch a graph of the path of the rocket.
 - How far from the wall is the rocket when it lands on the ground?
 - What is the maximum height of the rocket, and how far, horizontally, is it from the wall at that moment?

9. Investigate the list of quadratic relations and make a conjecture relating the x -intercepts and the coordinates of the vertex. Test your conjecture with another example.

$$y = (x + 1)(x - 1)$$

$$y = (x - 4)(x + 4)$$

$$y = -(x + 7)(x - 7)$$

$$y = 2(x + 6)(x - 6)$$

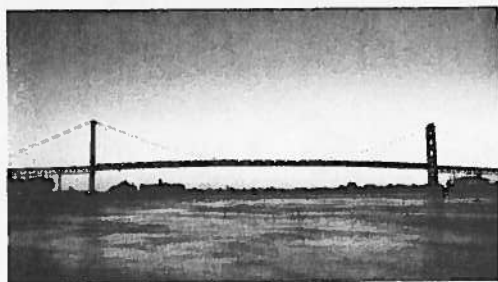
$$y = -\frac{1}{2}(x - 5)(x + 5)$$

10. A soccer ball is kicked from a point 23 m to the left of the halfway line and lands at a point 17 m to the right of the halfway line. It reaches a maximum height of 10 m during its parabolic flight.

- Sketch a graph to show the flight of the soccer ball if the halfway line passes through the origin.
- Determine an equation to represent the path of the soccer ball.

11. The Ambassador Bridge is a suspension bridge that crosses the Detroit River and connects Windsor, Ontario, to Detroit, Michigan. The two towers that support the centre span of cables rise 118 m above the river and are 564 m apart. The cable reaches its lowest point approximately 46 m above the river.

- Sketch a graph to show the curve of the cable if the origin is centred under the lowest point of the cable at the river's surface.
- Determine an equation to represent the curve of the cables in the form $y = a(x - r)(x - s)$, if possible. If not, explain why.



12. An architect designed the cross section of a museum using the quadratic relations

$$y = -\frac{1}{2}(x - 2)(x + 10), y = -(x + 4)(x - 4),$$

and $y = -1.5(x - 2)(x - 10)$, where y is the height, in metres, of the building, and x is the horizontal distance, in metres, from the middle of the building.

- Sketch a graph of the cross section of the museum.
- How wide is the entire museum?
- How tall is each section of the museum?

13. The revenue, R , from concert ticket sales at a local venue is calculated as (number of tickets sold) \times (price of ticket). The current price of a ticket is \$20, and the venue typically sells 100 tickets. For each \$1 increase in ticket price, 10 fewer tickets are sold. So, the revenue can be modelled using the equation $R = (100 - 10x)(20 + x)$, where x represents the number of \$1 increases.

- Rewrite the equation in the form $R = a(x - r)(x - s)$.
- Sketch a graph of the relation.
- What does the R -intercept represent? What do the x -intercepts represent?
- What does a negative value of x represent?
- What price maximizes the revenue?

Extend

14. **Use Technology** Use a graphing calculator to graph each relation. Describe and justify how the shape of the graph relates to the number of factors.

- $y = (x - 2)(x - 5)(x - 7)$

- $y = 2(x + 6)(x + 3)(x - 1)(x - 9)$

- $y = -x(x + 2)(x - 4)(x - 6)(x - 9)$

15. A point lies 2 units above the x -axis on the axis of symmetry of the parabola with equation $y = (x + 4)(x - 6)$. How far is it from this point to the point on the curve where $x = 5$?